

Some of the effect we see from the IIR filter design, such as the zeros at 1 or -1 being a small circle of points, instead of being at one location, is an artifact, not of the design process, but of representing these as a polynomial. To demonstrate this, the following MatLab script will create a polynomial with 21 zeros at 1. Then the zplane plot of this shows the location of the roots of the resulting polynomial. Also the roots are computed using roots.

```

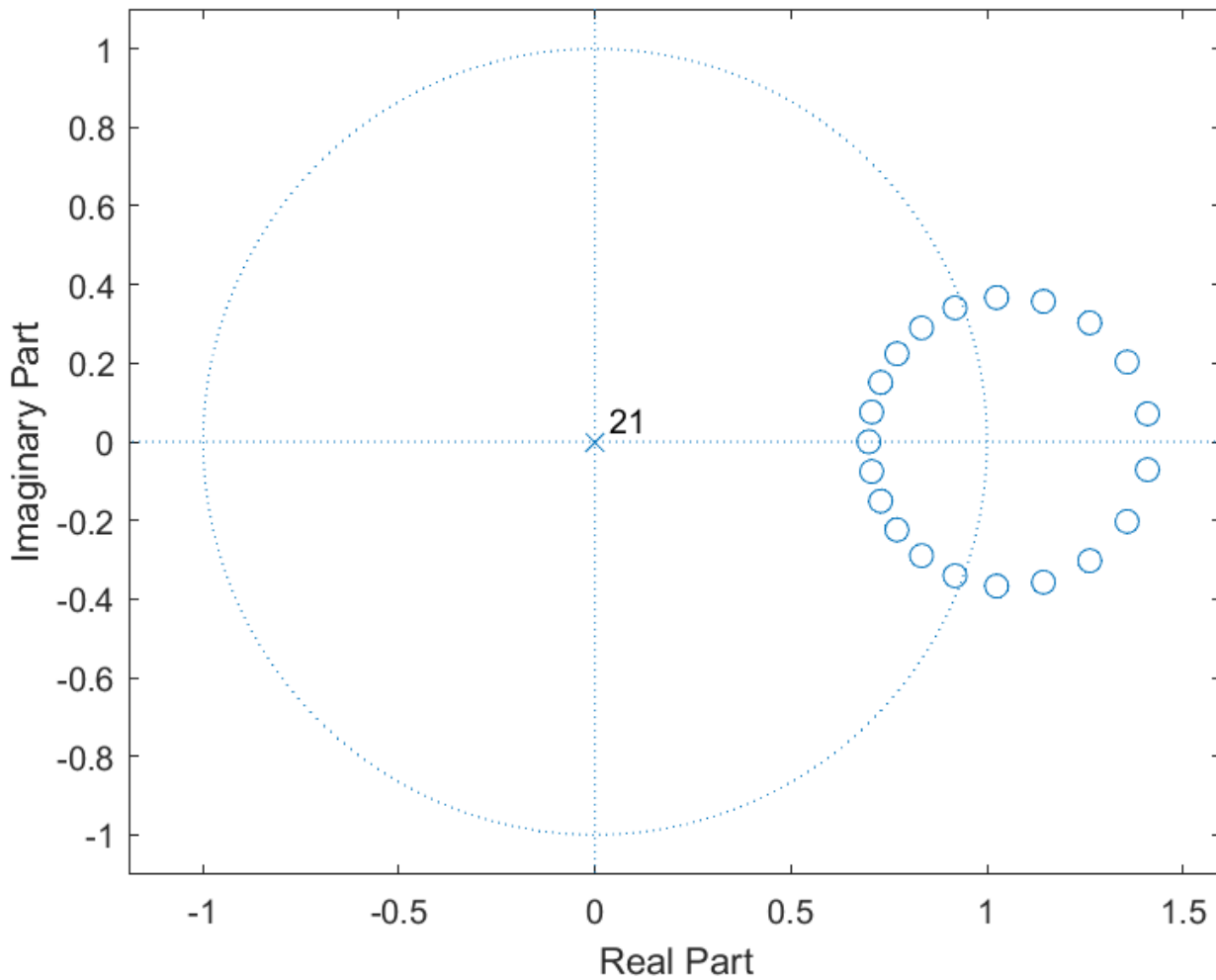
z = [1 -1]; % polynomial with root at 1.

z2 = z; % z2 will start as one root at 1.
for k = 1:20 % then we multiply the polynomial z2 with z 20 times.
    z2 = conv( z2, z );
end
% Next print out the coefficients of z2, noting that they don't look odd.
z2
%
% z2 =
% Columns 1 through 6
%      1      -21      210      -1330      5985      -20349
% Columns 7 through 12
% 54264  -116280  203490  -293930  352716  -352716
% Columns 13 through 18
% 293930  -203490  116280  -54264  20349  -5985
% Columns 19 through 22
%      1330      -210      21      -1

% With z2 being our 21'st order polynomial with 21 roots at 1
% we look at the roots of this polynomial
r = roots( z2 )
% r =
% 1.4092 + 0.0711i
% 1.4092 - 0.0711i
% 1.3571 + 0.2027i
% 1.3571 - 0.2027i
% 1.2619 + 0.3026i
% 1.2619 - 0.3026i
% 1.1438 + 0.3572i
% 1.1438 - 0.3572i
% 1.0243 + 0.3669i
% 1.0243 - 0.3669i
% 0.9182 + 0.3407i
% 0.9182 - 0.3407i
% 0.8327 + 0.2897i
% 0.8327 - 0.2897i
% 0.7699 + 0.2239i
% 0.7699 - 0.2239i
% 0.7284 + 0.1509i
% 0.7284 - 0.1509i
% 0.7054 + 0.0756i
% 0.7054 - 0.0756i
% 0.6981 + 0.0000i

% we display the roots in a zplane plot.
zplane( z2, 1 )

```



The 21 zeros are in a circle around +1. Caused by the rounding error of going from a large polynomial to roots.

The following script uses the same calls “butter” and “cheby1”, but requests three results which will be returned as a set of poles and zeros and a gain, rather than a polynomial.

Another aspect of the script is that all the filters are converted to Second Order Systems (sos), which is the most common form to implementing the filters.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Script to demonstrate pole-zero designs.
%
% Author: Dwight Day
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear
close all;

% Build impulse for generating impulse responses
Impulse = zeros( 250 , 1 );
Impulse(1) = 1.0;

% Define basic parameters.
Fs = 5000; % fs/2 for fs = 10K
% Low Pass
lp_pb = 500;
lp_sb = 600;
% Band Pass
bp_lsb = 500;
bp_lpb = 600;
bp_hpb = 1500;
bp_hsb = 2000;
% High Pass
hp_pb = 2000;
hp_sb = 1500;

%% IIR Filters B_xp, A_xp for each LowPass, BandPass and HighPass.

% low pass
[ nb , wb ] = buttord( lp_pb / Fs , lp_sb / Fs , 0.9 , 30 );
[ nz_lp,dz_lp ] = butter( nb , wb , 'low' );
% Repeat call, but request three results.
[ z_lp,p_lp,g_lp ] = butter( nb , wb , 'low' );
% Convert the zero pole form.
sos_lp = zp2sos( z_lp,p_lp,g_lp );

% band pass
[ nb , wb ] = buttord( [bp_lpb bp_hpb] / Fs , ...
                    [bp_lsb bp_hsb] / Fs , 0.9 , 30 );
[ nz_bp,dz_bp ] = butter( nb , wb );
[ z_bp,p_bp,g_bp ] = butter( nb , wb );
sos_bp = zp2sos( z_bp,p_bp,g_bp );

% high pass
[ nb , wb ] = buttord( hp_pb / Fs , hp_sb / Fs , 0.9 , 30 );
[ nz_hp,dz_hp ] = butter( nb , wb , 'high' );
[ z_hp,p_hp,g_hp ] = butter( nb , wb , 'high' );
sos_hp = zp2sos( z_hp,p_hp,g_hp);

% Plot frequency Response for low pass
[H,w] = freqz( sos_lp, [0:Fs*1000] , Fs*1000*2 );
ir = sosfilt( sos_lp , Impulse );
figure;
plot(w/1000,abs(H),...
     [0 lp_pb lp_pb 0 0], [0.9 0.9 1 1 0.9], 'r-',...
     [lp_sb lp_sb Fs] , [0.1 0.03 0.03], 'r-');
grid on;
title('Low Pass IIR: Frequency Response');
xlabel('Frequency (kHz)');
ylabel('Magnitude');
xlim([0 Fs]);
ylim([0 1.1]);
print -dpng LowPassIIR_FR.png

```

```

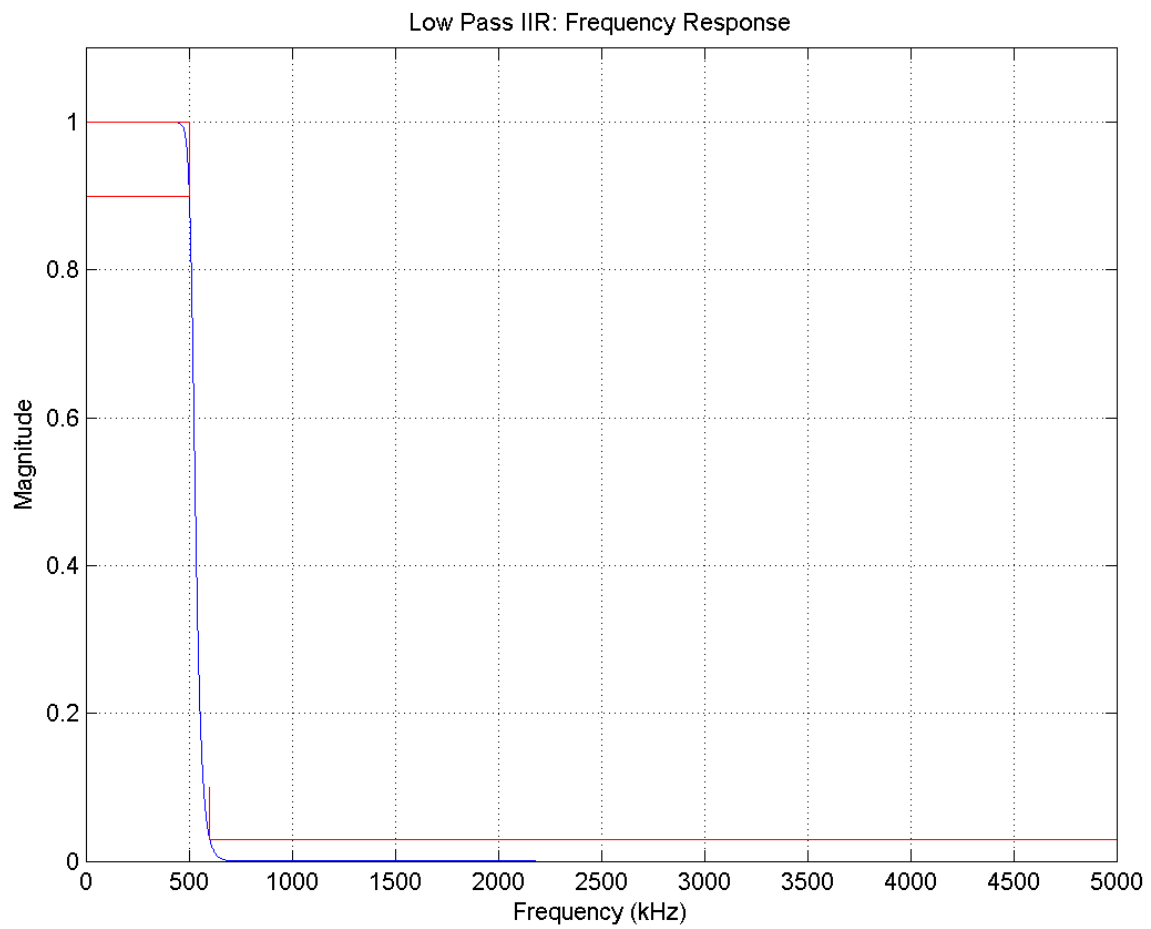
% pole-zero plot using the zero pole form.
figure;
subplot(211), zplane(z_lp,p_lp);
title('Low Pass IIR: Pole-Zero Plot');
grid on;
% Repeat pole-zero plot, based on polynomial form.
subplot(212), zplane(nz_lp,dz_lp);
print -dpng LowPassIIR_PZ.png
figure;
plot(ir);
title('Impulse Response - IIR Low Pass');
grid on;
print -dpng LowPassIIR_IR.png

% Repeat for band pass.
[H,w] = freqz( sos_bp, [0:Fs*1000] , Fs*1000*2 );
ir = sosfilt( sos_bp, Impulse );
% Plot frequency Response
figure;
plot(w/1000,abs(H),...
     [0 bp_lsb bp_lsb] , [0.03 0.03 0.1], 'r-',...
     [bp_lpb bp_hpb bp_hpb bp_lpb bp_lpb], [0.9 0.9 1 1 0.9], 'r-',...
     [bp_hsb bp_hsb Fs] , [0.1 0.03 0.03], 'r-');
grid on;
title('Band Pass IIR: Frequency Response');
xlabel('Frequency (kHz)');
ylabel('Magnitude');
xlim([0 Fs]);
ylim([0 1.1]);
print -dpng BandPassIIR_FR.png
% Plot Pole-Zero plot
figure;
subplot(211), zplane(z_bp,p_bp);
title('Band Pass IIR: Pole-Zero Plot');
subplot(212), zplane(nz_bp,dz_bp);
grid on;
print -dpng BandPassIIR_PZ.png
figure;
plot(ir);
title('Impulse Response - IIR Band Pass');
grid on;
print -dpng BandPassIIR_IR.png

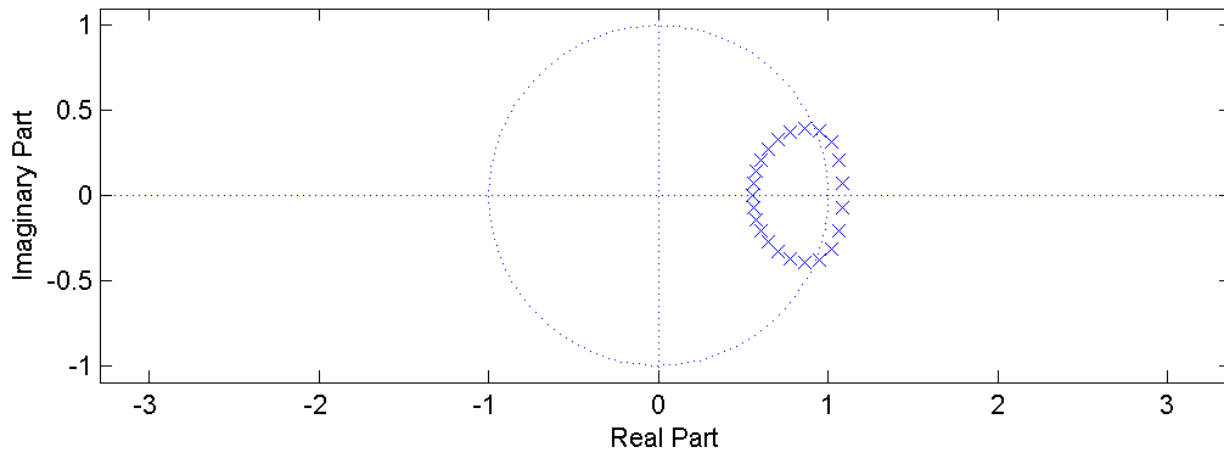
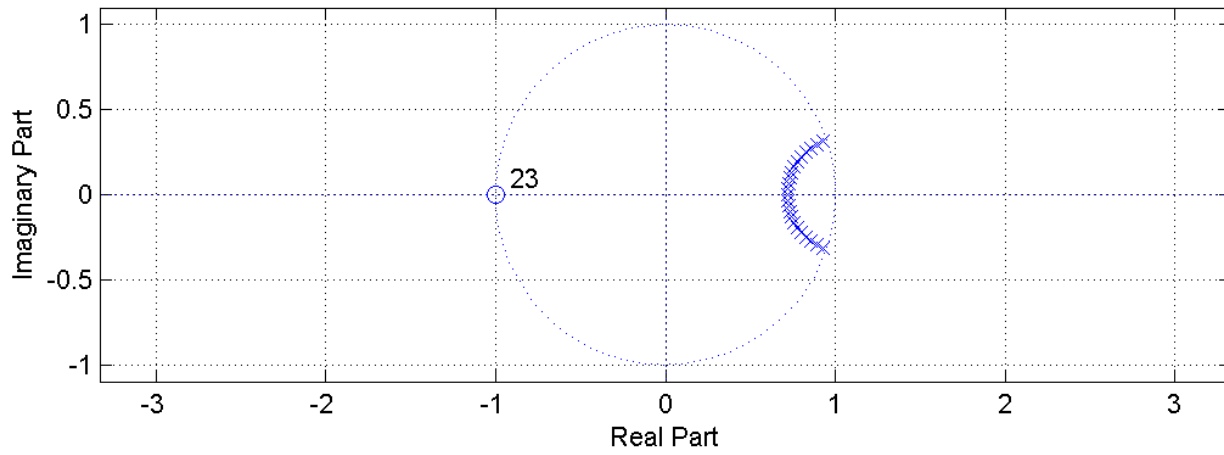
% Repeat for high pass.
[H,w] = freqz( sos_hp, [0:Fs*1000] , Fs*1000*2 );
ir = sosfilt( sos_hp , Impulse );
% Plot frequency Response
figure;
plot(w/1000,abs(H),...
     [0 hp_sb hp_sb] , [0.03 0.03 0.1], 'r-',...
     [hp_pb Fs Fs hp_pb hp_pb], [0.9 0.9 1 1 0.9], 'r-');
grid on;
title('High Pass IIR: Frequency Response');
xlabel('Frequency (kHz)');
ylabel('Magnitude');
xlim([0 Fs]);
ylim([0 1.1]);
print -dpng HighPassIIR_FR.png
% Plot Pole-Zero plot
figure;
subplot(212), zplane(z_hp,p_hp);
title('High Pass IIR: Pole-Zero Plot');
grid on;
subplot(211), zplane(nz_hp,dz_hp);
print -dpng HighPassIIR_PZ.png

```

```
figure;  
plot(ir);  
title('Impulse Response - IIR High Pass');  
grid on;  
print -dpng HighPassIIR_IR.png
```



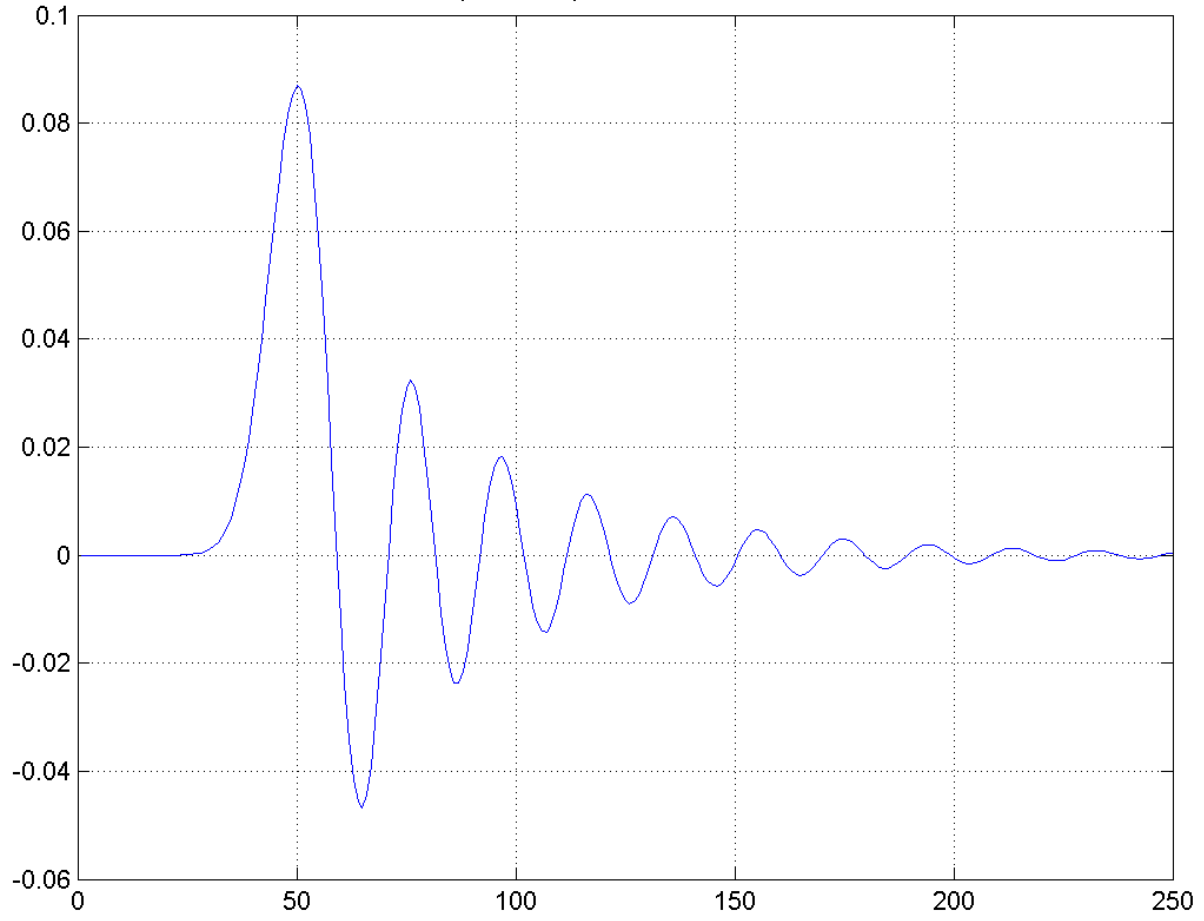
Low Pass IIR: Pole-Zero Plot



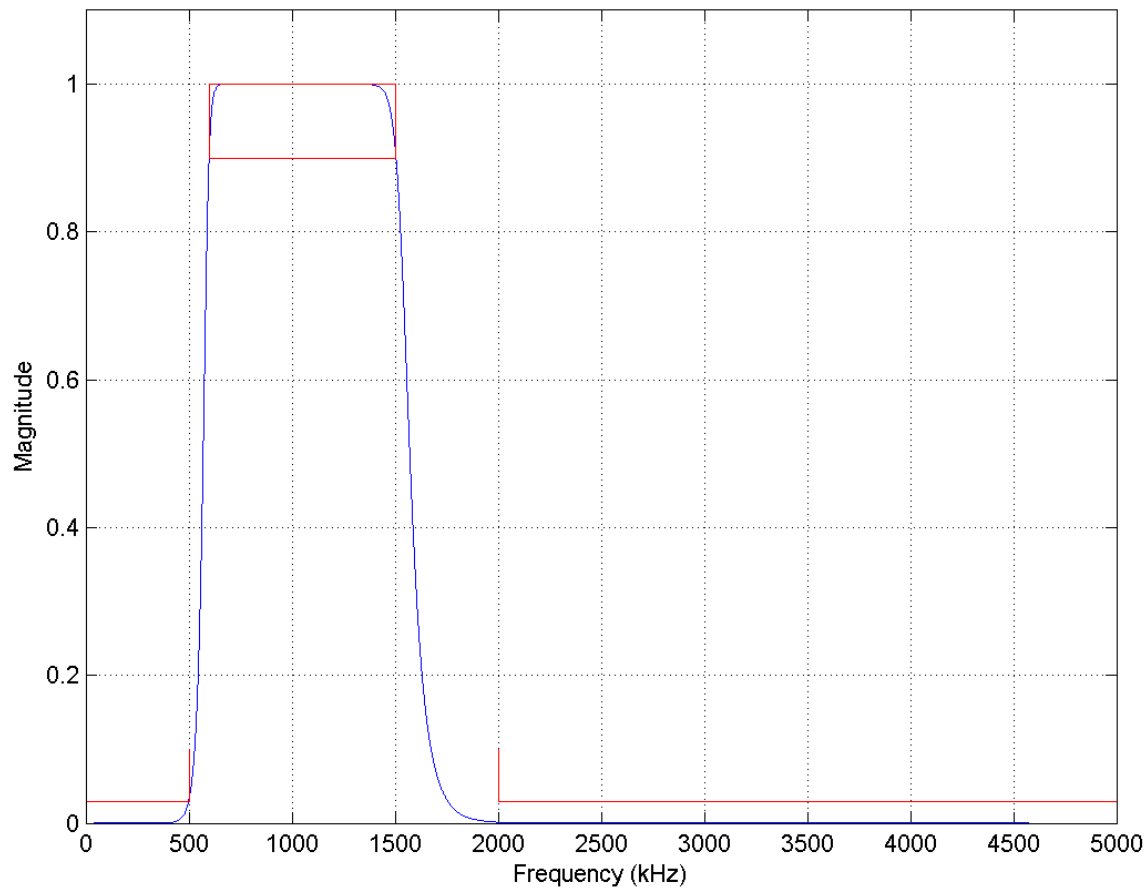
sos_lp =

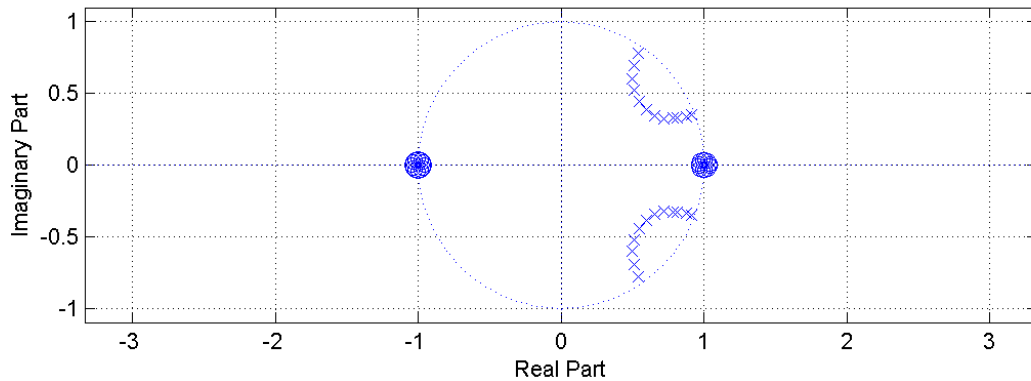
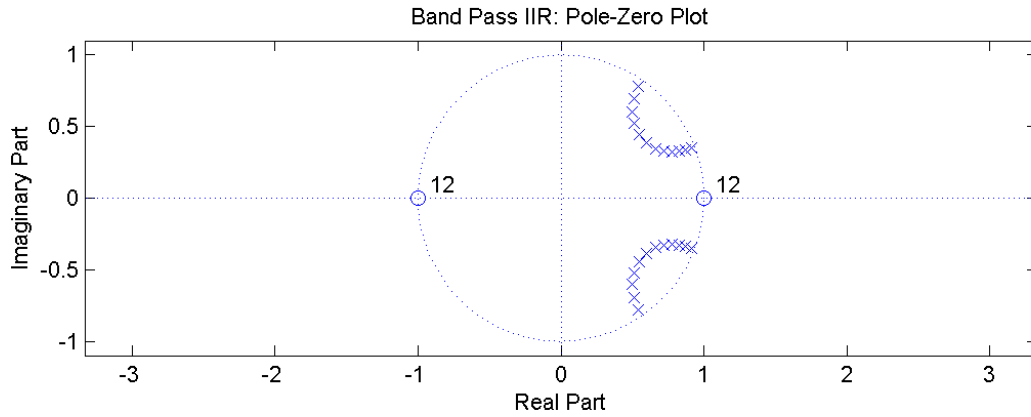
0.0000	0.0000	0	1.0000	-0.7180	0
1.0000	2.0000	1.0000	1.0000	-1.4392	0.5189
1.0000	2.0000	1.0000	1.0000	-1.4490	0.5292
1.0000	2.0000	1.0000	1.0000	-1.4653	0.5465
1.0000	2.0000	1.0000	1.0000	-1.4884	0.5709
1.0000	2.0000	1.0000	1.0000	-1.5184	0.6026
1.0000	2.0000	1.0000	1.0000	-1.5556	0.6417
1.0000	2.0000	1.0000	1.0000	-1.6000	0.6887
1.0000	2.0000	1.0000	1.0000	-1.6520	0.7435
1.0000	2.0000	1.0000	1.0000	-1.7118	0.8066
1.0000	2.0000	1.0000	1.0000	-1.7793	0.8779
1.0000	2.0000	1.0000	1.0000	-1.8546	0.9573

Impulse Response - IIR Low Pass



Band Pass IIR: Frequency Response





sos_bp =

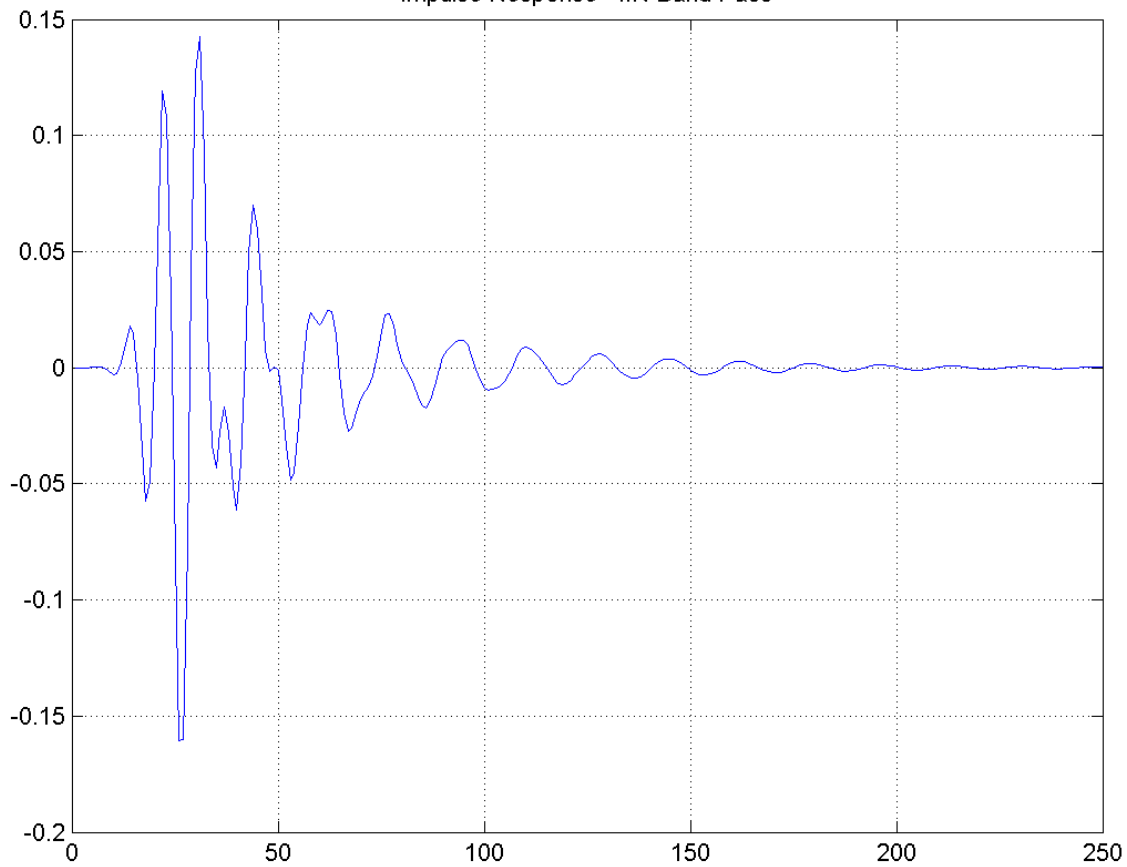
Columns 1 through 3

0.000000077965576	0.000000155931153	0.000000077965576
1.000000000000000	2.000000000000000	1.000000000000000
1.000000000000000	2.000000000000000	1.000000000000000
1.000000000000000	2.000000000000000	1.000000000000000
1.000000000000000	2.000000000000000	1.000000000000000
1.000000000000000	2.000000000000000	1.000000000000000
1.000000000000000	2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000

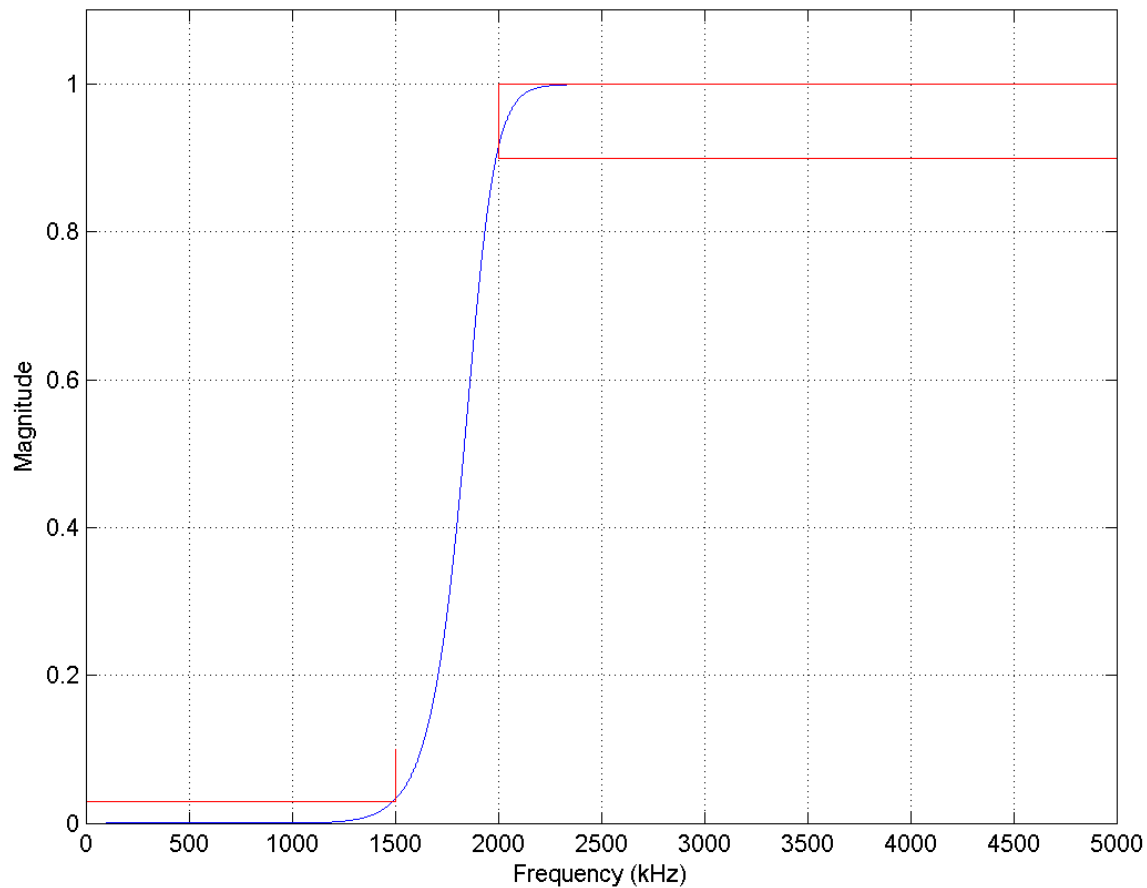
Columns 4 through 6

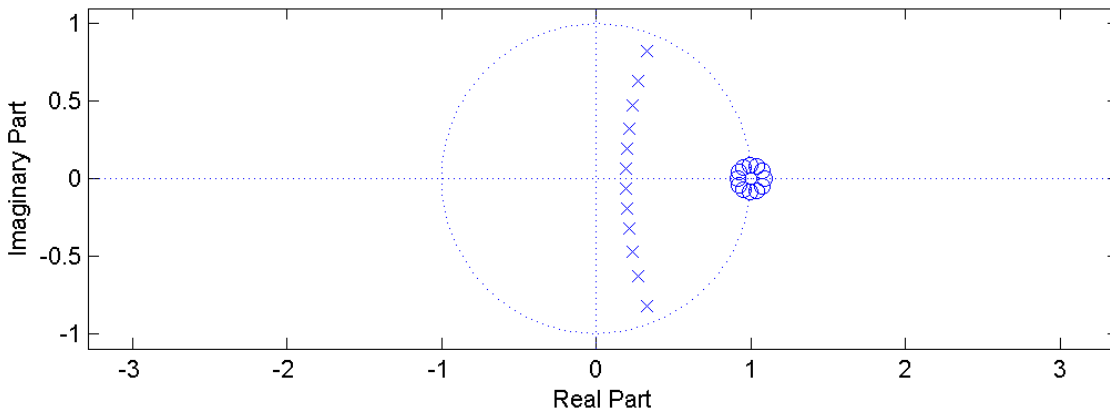
1.000000000000000	-1.091212035858335	0.496377485419364
1.000000000000000	-1.193797322320907	0.504600681780886
1.000000000000000	-1.025712450977045	0.534341024904205
1.000000000000000	-1.319055184538234	0.554770637667806
1.000000000000000	-1.000594518533117	0.615354124271003
1.000000000000000	-1.443452852837029	0.628818776769853
1.000000000000000	-1.554356870305770	0.709557826421234
1.000000000000000	-1.017590606573923	0.737597765720696
1.000000000000000	-1.652220084920662	0.790624059774055
1.000000000000000	-1.741375826371730	0.872003548379539
1.000000000000000	-1.081164862428158	0.901768176677095
1.000000000000000	-1.826111867242748	0.955994562870398

Impulse Response - IIR Band Pass

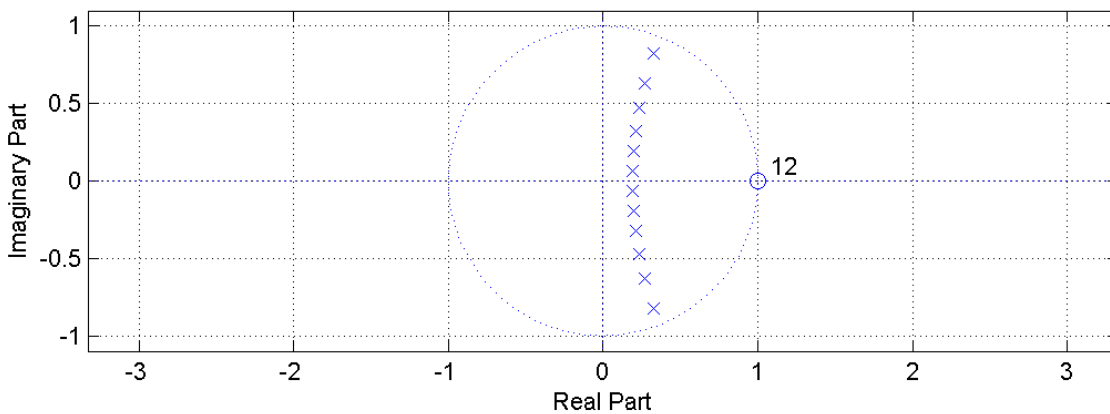


High Pass IIR: Frequency Response





High Pass IIR: Pole-Zero Plot



sos_hp =

Columns 1 through 3

0.006940032830899	-0.013880065661797	0.006940032830899
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000
1.000000000000000	-2.000000000000000	1.000000000000000

Columns 4 through 6

1.000000000000000	-0.383333689938468	0.040723542565666
1.000000000000000	-0.396286943997098	0.075890700593551
1.000000000000000	-0.423962960704219	0.151029105872549
1.000000000000000	-0.470425111983001	0.277170522458651
1.000000000000000	-0.543354059068179	0.475167396091769

Impulse Response - IIR High Pass

